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$SL(4)$ as a Common Symmetry Group of Electromagnetic and Gravitational Interactions

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Abstract

A theory of electromagnetic and gravitational interactions between elementary particles is proposed, based upon the symmetry group $SL(4)$. According to this theory, gravitational fields are formed as the result of interactions between charged particles with opposite charge.

1. *Introduction*

Electromagnetic and gravitational interactions have the Coulomb attraction in common. This indicates the possibility of constructing a theory that comprises both of these interactions. Many theories have been constructed on this subject, and this paper presents another one. However, our theory is based upon the application of a symmetry group.

In recent years, the application of symmetry groups in the theory of elementary particles has shown remarkable results. It is the purpose of this paper to show that both gravitational and electromagnetic interactions between elementary particles are governed by the same linear symmetry group.

Notation. Greek indices μ, ν, ρ, σ, α run from 1 to 4. Latin indices i, j, k run from 1 to 3. The summation convention is applied. Space-time coordinates are denoted by x^μ. x^4 is the time coordinate, $\mathbf{x}$ is the spatial coordinate vector. $\theta(x)$ is the step function defined by $\theta(x) = 1$ if $x > 0$, $\theta(x) = 0$ if $x < 0$.

2. *Some Irreducible Representations of* $SL(4)$

By $SL(4)$ we understand the group of real, linear, and homogenous transformations in four dimensions. The representatives $A^\mu{}_\nu$ of the infinitesimal generators satisfy the well-known commutation relations

$$[A^\mu{}_\nu, A^\rho{}_\sigma] = A^\mu{}_\sigma \delta^\rho{}_\nu - A^\rho{}_\nu \delta^\mu{}_\sigma \tag{2.1}$$

where $A^\mu{}_\nu$ are real matrices. The group invariants are given by

$$J_1 = A^\mu{}_\mu, \qquad J_2 = A^\mu{}_\nu A^\nu{}_\mu \tag{2.2}$$

An irreducible representation of the algebra (2.1) is given by the set of 4 x 4 matrices $A^\mu{}_\nu$, where the matrix element in the μth row and νth column is equal to unity, all other elements being zero. The column vectors u_μ that form the bases, transform like covariant four-vectors, and we denote this representation by $\overline{IR}_4$.

By means of the transformation $A^\mu{}_\nu \to -\tilde{A}^\mu{}_\nu$ (matrix $\tilde{A}$ is the transposition of A), we obtain another irreducible representation. The column vectors u^μ that form the bases of this representation transform like contravariant four-vectors, and we denote this representation by IR_4. The 16-dimensional representation space of the product $IR_4 \times IR_4$ has two irreducible subspaces with 6 and 10 dimensions:

$$IR_4 \times IR_4 = IR_6 + IR_{10} \tag{2.3}$$

The components of the column vectors that form the bases of IR_6 and IR_{10} transform, respectively, like antisymmetric and symmetric contravariant tensors of order 2.

Similarly we have

$$\overline{IR}_4 \times \overline{IR}_4 = \overline{IR}_6 + \overline{IR}_{10} \tag{2.4}$$

The components of the column vectors that form the bases of $\overline{IR}_6$ and $\overline{IR}_{10}$ transform, respectively, like antisymmetric and symmetric covariant tensors of order 2.

The product space $IR_4 \times \overline{IR}_4$ has two irreducible subspaces with 1 and 15 dimensions:

$$IR_4 \times \overline{IR}_4 = IR_1 + IR^M_{15} \tag{2.5}$$

The components of the column vectors that form the bases of IR^M_{15} transform like mixed tensors of order 2.

The product $\overline{IR}_6 \times IR_4$ has two irreducible subspaces with 4 and 20 dimensions:

$$\overline{IR}_6 \times IR_4 = IR_4 + IR^M_{20} \tag{2.6}$$

The column vectors that form the bases of IR^M_{20} have components that are mixed tensors of order 3, with the symmetry property

$$t^\sigma_{\mu\nu} = -t^\sigma_{\nu\mu} \tag{2.7}$$

$SL(4)$ has also an infinite dimensional irreducible representation. By means of the operators X^μ, Y_ν that satisfy the Heisenberg algebra

$$[Y_\nu, X^\mu] = \delta^\mu{}_\nu, \qquad [X^\mu, X^\nu] = 0 = [Y_\mu, Y_\nu] \tag{2.8}$$

we construct the operators

$$D^\mu{}_\nu = X^\mu Y_\nu \tag{2.9}$$

that satisfy the algebra (2.1).

By the help of (2.8) we get the following relation between the invariants:

$$D^{\mu}{}_{\nu}D^{\nu}{}_{\mu} = D^{\mu}{}_{\mu}(3 + D^{\nu}{}_{\nu}) \tag{2.10}$$

In order to obtain the bases of an irreducible representation, we consider the vector $|a, \alpha\rangle$, $-\infty < a < \infty$, $\alpha = 1, 2$, belonging to the irreducible representation space of the Heisenberg algebra

$$[Y, X] = 1 \tag{2.11}$$

If $\langle x|$ is an eigenbra of the Hermitean operator X, corresponding to the eigenvalue x, $|a, \alpha\rangle$ is defined by

$$\langle x|a, 1\rangle = (2\pi)^{-1/2}\theta(x)e^{-(1/2+ia)\log x} \tag{2.12}$$

$$\langle x|a, 2\rangle = (2\pi)^{-1/2}\theta(-x)e^{-(1/2+ia)\log(-x)} \tag{2.12'}$$

By straightforward calculations we verify the relations

$$\langle a', \alpha'|a, \alpha\rangle = \delta_{\alpha\alpha'}\delta(a - a'), \qquad \sum_{\alpha=1}^{2}\int_{-\infty}^{\infty} da\langle x'|a, \alpha\rangle\langle a, \alpha|x\rangle = \delta(x - x') \tag{2.13}$$

The vectors $|a, \alpha\rangle$ are thus bases of an irreducible representation of the Heisenberg algebra (2.11).

Denote by $|b, \beta\rangle$ the direct product of four vectors of the type $|a, \alpha\rangle$. The vectors $|b, \beta\rangle$ form the bases of an irreducible representation of the Heisenberg algebra (2.8). By the help of the formula

$$\langle x|D^{\mu}{}_{\nu} = x^{\mu}\acute{\partial}_{\nu}\langle x| \tag{2.14}$$

we get the equation

$$D^{\mu}{}_{\mu}|b, \beta\rangle = \left(-2 + i\sum_{\mu=1}^{4} b_{\mu}\right)|b, \beta\rangle \tag{2.15}$$

The invariants of $SL(4)$ are real numbers, and consequently the vectors

$$|b, \beta\rangle, \quad \sum_{\mu=1}^{4} b_{\mu} = 0 \tag{2.16}$$

form the bases of an infinite-dimensional irreducible representation of $SL(4)$.

With $D^{\mu}{}_{\nu} = x^{\mu}\partial_{\nu}$, and the bases defined by the functions $\langle x|b, \beta\rangle$, we shall denote this representation by $IR(x)$. Because of the relation (2.10), we see that both invariants have the value -2 in this representation.

The representation formed by the product $IR_N \times IR(x)$, where IR_N is a finite-dimensional irreducible representation, has infinitesimal generators

$$D^{\mu}{}_{\nu} + A^{\mu}{}_{\nu}$$

The invariant $J_1 = D^{\mu}{}_{\mu} + A^{\mu}{}_{\mu}$ is a constant multiple of the unit matrix within the whole product space. But $J_2 = (D^{\mu}{}_{\nu} + A^{\mu}{}_{\nu})(D^{\nu}{}_{\mu} + A^{\nu}{}_{\mu})$ is a constant multiple of the unit matrix only within subspaces defined by

$$(D^{\mu}{}_{\nu}A^{\nu}{}_{\mu} + n)\,|\phi\rangle = 0 \tag{2.17}$$

where n is a number that depends upon the value of $A^{\mu}{}_{\mu}$. For real values of n, the subspace defined by (2.17) is irreducible under $SL(4)$, provided that $|\phi\rangle$ also satisfies the condition

$$(D^{\mu}{}_{\mu} + 2)|\phi\rangle = 0 \tag{2.17'}$$

The components of $\langle x|\phi\rangle$ transform like tensors, and in component form, (2.17) form a set of tensor equations.

Making use of the explicit matrices $A^{\mu}{}_{\nu}$ belonging to $\overline{IR}_4$, we find that the component form of (2.17) is given by

$$x^{\nu}\partial_{\mu}\phi_{\nu} + n\phi_{\mu} = 0$$

If $A^{\mu}{}_{\nu}$ belong to IR_4, the component form of (2.17) is given by

$$-x^{\mu}\partial_{\nu}\phi^{\nu} + n\phi^{\mu} = 0$$

If the matrices $A^{\mu}{}_{\nu}$ belong to IR_6 or IR_{10}, the components of $\langle x|\phi\rangle$ are contravariant tensors of order 2, and (2.17) has the component form

$$-x^{\mu}\partial_{\sigma}\phi^{\sigma\nu} - x^{\nu}\partial_{\sigma}\phi^{\mu\sigma} + n\phi^{\mu\nu} = 0$$

If $A^{\mu}{}_{\nu}$ belong to $\overline{IR}_6$ or $\overline{IR}_{10}$, the components of $\langle x|\phi\rangle$ are covariant tensors of order 2, and (2.17) has the component form

$$x^{\sigma}\partial_{\mu}\phi_{\sigma\nu} + x^{\sigma}\partial_{\nu}\phi_{\mu\sigma} + n\phi_{\mu\nu} = 0$$

In the following applications, the value of n equals zero in the contravariant equations, and it equals 2 in the covariant and mixed equations.

The component equations of (2.17) that are of interest, are given by

$$\partial_{\mu}\phi^{\mu} = 0 \tag{2.18}$$

$$x^{\nu}\partial_{\mu}\phi_{\nu} + 2\phi_{\mu} = 0 \tag{2.19}$$

$$\partial_{\mu}\phi^{\mu\nu} = \partial_{\nu}\phi^{\mu\nu} = 0 \tag{2.20}$$

$$x^{\sigma}\partial_{\mu}\phi_{\sigma\nu} + x^{\sigma}\partial_{\nu}\phi_{\mu\sigma} + 2\phi_{\mu\nu} = 0 \tag{2.21}$$

$$x^{\sigma}\partial_{\mu}\phi^{\rho}_{\sigma\nu} + x^{\sigma}\partial_{\nu}\phi^{\rho}_{\mu\sigma} - x^{\rho}\partial_{\sigma}\phi^{\sigma}_{\mu\nu} + 2\phi^{\rho}_{\mu\nu} = 0 \tag{2.22}$$

In addition we have (2.17′), which we write in the form

$$(x^{\mu}\partial_{\mu} + 2)\phi = 0 \tag{2.23}$$

3. *Particles and Antiparticles*

As the representative of a charged particle, we shall use the column vector U, with components U^{μ} that transform under IR_4. In the standard state (rest state) the components of U are given by

$$U_{\pm} = (0, 0, 0, U_{\pm}^4) = (0, 0, 0, \pm 1) \tag{3.1}$$

where U_+ is a particle with positive charge, and U_- is a particle with negative charge.

We shall assume that a neutral particle might be formed as the result of an interaction between U_+ and U_-. A possible neutral particle state is thus given by

$$t = \xi U_+ U_-$$

where ξ is an interaction constant.

We write

$$t = s + a$$

where the components $s^{\mu\nu} = s^{\nu\mu}$ of s transform under IR_{10}, and the components $a^{\mu\nu} = -a^{\nu\mu}$ of a transform under IR_6. s is interpreted as a representative of a neutral massive particle, and the standard state is given by

$$s^{i\mu} = 0, \qquad s^{44} = 1 \tag{3.2}$$

A neutral zero mass particle, such as the photon, is supposed to be represented by a.

A charged antiparticle is represented by a column vector $\overline{U}$, with components U_μ that transform under $\overline{IR}_4$. Neutral antiparticles might be represented by the column vectors $\bar{s}$ and $\bar{a}$, that transform under $\overline{IR}_{10}$ and $\overline{IR}_6$, respectively.

However, a neutral particle might also be formed as the result of interactions between particles and antiparticles, and the column vector w, with components $w^\mu{}_\nu$ that transform under IR_{15}^M, is also a possible representative of a neutral particle. The standard state of a massive particle is given by

$$w^i{}_\mu \mp 0 = w^\nu{}_i, \qquad w^4{}_4 = 1 \tag{3.3}$$

The neutron is supposed to be of the type (3.3), because it decays into a proton, an electron, and an antineutrino. The neutron state is therefore of the form $U_+ U_- \bar{a}$.

4. *Photons*

Define the functions η^μ by

$$\eta^i = |\mathbf{x}|^{-3} x^i, \qquad \eta^4 = 0 \tag{4.1}$$

η is a static, spherically symmetric field with center at the origin. Since the center is not moving, (4.1) is chosen as the standard state of this field. Function (4.1) is a solution of (2.18) and (2.23) and therefore transforms under $IR_4(x)$.

The column vector $\bar{\eta}$ with components

$$\eta_\mu = (\eta^i, 0) \tag{4.2}$$

transforms under $\overline{IR}_4(x)$, since (4.2) is a solution of (2.19) and (2.23). We shall interpret

$$\phi = k\eta, \qquad \bar{\phi} = k\bar{\eta}$$

as photon fields in the vacuum state; k is a constant.

Let us consider an interaction between the vacuum state photon ϕ and a charged particle U. The interaction state is represented by the column vector

$$\phi U = c_6\psi_6 + c_{10}\psi_{10}$$

The components of ψ_6 have the form

$$\psi_6^{\mu\nu} = \epsilon(\phi^\mu U^\nu - \varphi^\nu U^\mu) \tag{4.3}$$

where ϵ is an interaction constant.

For arbitrary values of ϕ^μ and U^ν, the components $\psi_6^{\mu\nu}$ do not transform among themselves. However, if ϕ and U are the standard states (4.1) and (3.1), respectively, (4.3) is given by

$$\Phi^{ij} = 0, \qquad \Phi^{i4} = \epsilon\phi^i \tag{4.4}$$

which is a solution of (2.20).

Similarly, an interaction $\bar{\phi}\bar{U}$ forms the field $\bar{\Phi}$, which in the standard state is given by

$$\Phi_{ij} = 0, \qquad \Phi_{i4} = \epsilon\varphi^i \tag{4.5}$$

$\Phi_{\mu\nu}$ is a solution of (2.21) and transforms thus under $\overline{IR}_6(x)$. The dual of $\Phi_{\mu\nu}$, $\Phi^{*\mu\nu} = \epsilon^{\mu\nu\rho\sigma}\Phi_{\rho\sigma}$, is a solution of (2.20), and the dual of $\Phi^{\mu\nu}$, $\Phi^*_{\mu\nu} = \epsilon_{\mu\nu\rho\sigma}\Phi^{\rho\sigma}$, is a solution of (2.21).

The fact that $\Phi^{\mu\nu}$ and $\Phi^{*\mu\nu}$ are solutions of (2.20) is expressed by the Maxwell equations of electromagnetic fields in empty space. From the point of view of the present theory, the Maxwell equations state that $\Phi^{\mu\nu}$ and $\Phi^{*\mu\nu}$ transform under $IR_6(x)$.

Function (4.4) is interpreted as the photon field created by the particle U at rest at the origin. The interaction ϕU thus has the effect of raising the vacuum state photons ϕ into the observable state Φ. The field equations (2.20) and the "antifield" equations (2.21) have the same standard state solutions. This might be interpreted as expressing that photons and antiphotons are indistinguishable. The photon equations (2.18) and (2.19) have the same property. It is important to note that Φ and $\bar{\Phi}$, in the standard state, represent a field of photons outside the origin, plus a charged particle at rest at the origin.

We consider now an interaction between the system $\bar{\Phi}$ and a particle U, where U is assumed to have the opposite charge of the particle at the center of the field $\bar{\Phi}$. The interaction state is given by

$$\bar{\Phi}U = c_4\psi_4 + c_{20}\psi_{20} \tag{4.6}$$

where c_4 and c_{20} are constants. The components of ψ_4 are given by

$$\alpha\Phi_{\mu\nu}U^\nu \tag{4.7}$$

where α is a constant. For arbitrary values of $\Phi_{\mu\nu}$ and U^ν, ψ_4 do not transform under an irreducible representation of $SL(4)$. In this case (4.7) has the familiar interpretation as the change in four-momentum with proper time, of the particle U. Of course, this interpretation is possible only as long as the particle state U is connected with the standard state (3.1) by a Lorentz transformation.

If $\bar{\Phi}$ and U are standard states, ψ_4 is proportional to $\bar{\phi}$ and transforms under $\overline{IR}_4(x)$. This is interpreted as the particular interaction between particles of opposite charge that results in the formation of a neutral particle. In this interaction, the field $\bar{\Phi}$ is converted into the vacuum state field $\bar{\phi}$.

5. *Gravitons and Gravitational Interactions*

We consider now the state ψ_{20}, which is the other possible outcome of the interaction (4.6). The components of ψ_{20} are given by

$$\Gamma^\sigma_{\mu\nu} = \kappa\Phi_{\mu\nu}U^\sigma \tag{5.1}$$

If $\bar{\Phi}$ and U are standard states, the 20 components $\Gamma^\sigma_{\mu\nu}$ have the form

$$\Gamma^i_{\mu\nu} = 0, \qquad \Gamma^4_{i4} = GM\eta^i \tag{5.2}$$

where the interaction constant κ is written $\kappa = (\epsilon k)^{-1}GM$. Equations (5.2) form a solution of (2.22) and transform under $IR^M_{20}(x)$. The irreducible field state given by (5.2) is interpreted as the gravitational field of a particle that is at rest at the origin. M is the mass of the particle, and G is the Newtonian gravitational constant. We write

$$U\bar{\Phi} \to \Gamma \tag{5.3}$$

However, Γ might also be formed by an interaction between a neutral particle w and a vacuum state photon $\bar{\phi}$:

$$w\bar{\phi} \to \Gamma \tag{5.4}$$

The components of $w\bar{\phi}$ that form the field Γ are given by

$$\Gamma^\sigma_{\mu\nu} = k^{-1}GM(w^\sigma{}_\nu\phi_\mu - w^\sigma{}_\mu\phi_\nu) \tag{5.5}$$

With w and $\bar{\phi}$ in the standard state, (5.5) and (5.2) are identical. Let us consider an interaction between a neutral particle w and the field (5.2). We write

$$mw\Gamma \to \chi_4 \tag{5.6}$$

where the interaction constant m equals the mass of the particle w. We assume that the components of the interaction state vector χ_4 have the form

$$mw^\nu{}_\sigma\Gamma^\sigma_{\mu\nu} \tag{5.7}$$

Like (4.7) we interpret (5.7) as the change in four-momentum with proper time of the particle w; m is the mass of w:

$$mw^{\nu\sigma}\Gamma^{\sigma}_{\mu\nu} = \frac{dp_\mu}{d\tau} \tag{5.8}$$

If w is moving with velocity $\mathbf{v}$, we have $w^1{}_4 = \gamma^2 v^i$, $w^4{}_4 = \gamma^2$, $\gamma^2 = 1 - \mathbf{v}^2$. Making use of the relations

$$\frac{dt}{d\tau} = \gamma, \qquad \mathbf{p} = -(p_1, p_2, p_3)$$

(5.8) is written

$$\frac{d\mathbf{p}}{dt} = -GMm\gamma|\mathbf{x}|^{-3}\mathbf{x} = \mathbf{F}, \qquad \frac{dp_4}{dt} = \mathbf{F}\cdot\mathbf{v} \tag{5.9}$$

For moderate velocities $\mathbf{v}$, where $\gamma \simeq 1$, the equations (5.9) are the Newtonian equations of motion of a particle in a gravitational field with source at the origin. Equation (5.8) expresses these equations in relativistic covariant form.

We have thus shown that our interpretation of (5.2) and (5.5) as a field of gravitons is reasonable.

Let us for a moment return to the electromagnetic interactions. An interaction between the Maxwell field Φ, Φ^* and a particle U is, up to a constant factor, given by $\Phi^* U$, with the components

$$\epsilon_{\mu\nu\rho\sigma}\Phi^{*\rho\sigma}U^\nu = \Phi_{\mu\nu}U^\nu$$

We therefore write

$$\Phi^* U = \bar{\Phi} U \tag{5.10}$$

Because photons are their own antiparticles, the interaction between the Maxwell field Φ^* and U equals the interaction between the "antifield" $\bar{\Phi}$ and U. But interactions between charged particles are most conveniently expressed by means of the "antifield" representation $\bar{\Phi}$. According to (5.4), the neutron creates a gravitational field Γ_n that in the standard state is given by (5.2), where M is the mass of the neutron. But a single proton or electron does not create gravitational fields, according to our theory. According to (5.3), a gravitational field is created as the result of an interaction between charged particles. The irreducible field state (5.2) occurs if the particle U is at rest with respect to the same referential as the particle that creates the field $\bar{\Phi}$. Physically, this can only be realized if the particle U has a charge that is opposite the charge of the particle that creates the field $\bar{\Phi}$.

We consider now a nucleus that is formed by two protons. This nucleus creates an electromagnetic field $2\bar{\Phi}_p$, where $\bar{\Phi}_p$ is the proton field. Suppose that the nucleus interacts with an electron U_e in such a way that the electron is absorbed. In this case the interaction (5.3) forms a gravitational field. Assuming that

$$U_e\bar{\Phi}_p \to \Gamma_n \tag{5.11}$$

this field is given by $2\Gamma_n$. The absorption of the electron converts one of the protons into a neutron, and the resulting nucleus is a deuteron. The gravitational field of the deuteron is thus $2\Gamma_n$. As far as gravitation is concerned, an arbitrary nucleus is considered to be composed of deuterons and neutrons. The α particle is thus composed of two deuterons and creates a gravitational field $4\Gamma_n$. In general, a nucleus with Z protons and $N - Z$ neutrons is considered to be composed of Z deuterons and $N - 2Z$ neutrons. The gravitational field created by this nucleus is therefore given by

$$Z2\Gamma_n + (N - 2Z)\Gamma_n = N\Gamma_n \tag{5.12}$$

Being obtained by simple addition, the field (5.12) is not expected to be quite accurate, because the nuclear binding energy causes small modifications in the mass.

Equations (5.9) show that the principle of equivalence is rigorously satisfied as far as neutral particles are concerned. But according to our theory, charged particles like protons and electrons violate this principle. It is of course impossible to decide experimentally whether a proton creates a gravitational field or not.

On the other hand, our theory does not exclude the possibility of an interaction between a proton or electron U and a gravitational field Γ. But the components of the interaction state vector ΓU are tensors of order 2, and this complicates the physical interpretation of this state.

The hydrogen atom represents an interaction state of the type (5.11), and if the oscillatory motion of the electron is neglected, the hydrogen atom creates a gravitational field of the order of magnitude Γ_n. If the motion of the electron is not neglected, the gravitational field of the hydrogen atom is still existing, but it is not static.

The tensor $\Gamma^{\mu\nu\sigma}$ that is a solution of the equations

$$\partial_\mu \Gamma^{\mu\nu\sigma} = \partial_\nu \Gamma^{\mu\nu\sigma} = \partial_\sigma \Gamma^{\mu\nu\sigma} = 0 \tag{5.13}$$

is also interpreted as a gravitational field. This field is supposed to be formed by the interactions

$$\Phi U \to \Gamma \tag{5.14}$$

$$\phi a \to \Gamma_0 \tag{5.15}$$

$$\phi s \to \Gamma \tag{5.16}$$

The state vector ϕa in (5.15) has the components $\phi a = c_4 \chi + c_{20}\Gamma_0$, where

$$\begin{aligned} \chi^{\mu\nu\sigma} &= (1/2c_4)(\phi^\mu a^{\nu\sigma} - \phi^\nu a^{\mu\sigma} - \phi^\sigma a^{\nu\mu}) \\ \Gamma_0^{\mu\nu\sigma} &= (1/2c_{20})(\phi^\mu a^{\nu\sigma} + \phi^\nu a^{\mu\sigma} + \phi^\sigma a^{\nu\mu}) \end{aligned} \tag{5.17}$$

If the four nonvanishing components $\chi^{\mu\nu\sigma}$ form a solution of (5.13), the field χ transforms under $\overline{IR}_4(x)$.

If the 20 components (5.17) form a solution of (5.13), the field Γ_0 transforms under $IR_{20}(x)$ and is interpreted as the gravitational field created by the zero-mass particle a.

Our theory thus predicts that both photons and neutrinos create gravitational fields.

As far as electromagnetic interactions are concerned, there is complete symmetry between particles and antiparticles. This is not so in the case of gravitational interactions. For instance, the gravitational field created by an antiproton-positron interaction $\bar{\Phi}_p \overline{U}_e (\Phi_p \overline{U}_e)$ has the components $\Gamma_{\mu\nu\sigma}(\Gamma_\sigma^{\mu\nu})$ In order to transform under an irreducible representation of $SL(4)$, these fields have to satisfy the equations

$$x^\alpha \partial_\mu \Gamma_{\alpha\nu\sigma} + x^\alpha \partial_\nu \Gamma_{\mu\alpha\sigma} + x^\alpha \partial_\sigma \Gamma_{\mu\nu\alpha} + n\Gamma_{\mu\nu\sigma} = 0 \tag{5.18}$$

$$-x^\mu \partial_\alpha \Gamma_\sigma^{\alpha\nu} - x^\nu \partial_\alpha \Gamma_\sigma^{\mu\alpha} + x^\alpha \partial_\sigma \Gamma_\alpha^{\mu\nu} + n\Gamma_\sigma^{\mu\nu} = 0 \tag{5.19}$$

But (5.18) and (5.19) have no static solutions of the type (5.2).

6. *Final Remarks*

In Section 3 we have defined particle and antiparticle states by means of the bases of some of the irreducible representations of $SL(4)$, which are mentioned in Section 2. In Section 4 we show that these definitions and interpretations lead to acceptable results as far as electromagnetic interactions are concerned. Of some interest is the interpretation of the Maxwell equations, as a necessary (but not sufficient) condition that the photon fields transform under an irreducible representation of $SL(4)$.

In Section 5, which we consider to be the important part of the paper, we have shown that it is a possibility that gravitons are formed by interactions between charged particles.